EXPANSION OF AN IDEALLY PLASTIC CYLINDRICAL SHELL IN RESPONSE TO DETONATION PRODUCTS

V. A. Odintsev, V. V. Selivanov, and L. A. Chudov

UDC 539.374.1

A numerical solution is presented for the nonlinear differential equation for the one-dimensional motion of an ideally plastic incompressible shell subject to pressure from a uniformly expanding gas. The stress distribution in the shell and the law of motion are derived. The failure radius is discussed.

There are various papers [1-5] on the behavior of an ideally plastic incompressible shell subject to pressure from detonation products; the state of strain is complex, and there are tension and compression zones, for which relationships have been derived to define the failure radius. There is no study in [1-5] of the kinematics and energy redistribution.

We consider the planar deformation of a cylindrical shell (Fig. 1) in response to products obeying the law

$$pV^k = \text{const} \tag{1}$$

where p and V are pressure and specific volume.

The stresses σ_r, σ_θ, and σ_z are principal ones. The internal and external initial radii are a_0 and b_0, while the current ones are a and b.

We assume that the shell material is incompressible and define the integral of the equation of continuity in the form

$$v = \dot{a}\, a / r \tag{2}$$

where r is an Euler coordinate, v is the radial velocity of a particle, and $\dot{a} = da/dt$ is the speed of the internal surface.

We substitute (2) and the derivatives $\partial v/\partial t$ and $\partial v/\partial r$ in the Euler equation

$$\gamma_0 \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = \frac{\partial \sigma_r}{\partial r} - \frac{\sigma_\theta - \sigma_r}{r}$$

where γ_0 is the density and used the plasticity condition

$$\sigma_\theta - \sigma_r = \varkappa Y \tag{3}$$

where Y is the dynamic yield point to get

$$\frac{\partial \sigma_r}{\partial r} = \frac{\varkappa Y}{r} + \gamma_0 \left(\frac{a\ddot{a} + \dot{a}^2}{r} - \frac{a^2 \dot{a}^2}{r^3} \right) \tag{4}$$

Here $\ddot{a} = d^2a/dt^2$ is the acceleration of the internal surface.

In (3), $\varkappa = 1$ if the St. Venant plasticity condition applies, and $\varkappa = 2/\sqrt{3}$ if the Mises plasticity condition applies (the deformation is planar, $\sigma_z = (\sigma_r + \sigma_0)/2$].

Moscow. Translated from Prikladnoi Matematiki i Tekhnicheskoi Fiziki, No. 2, pp. 152-156, February, 1974. Original article submitted August 27, 1973.

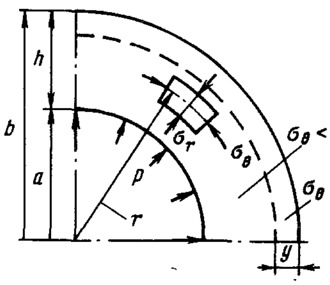

Fig. 1

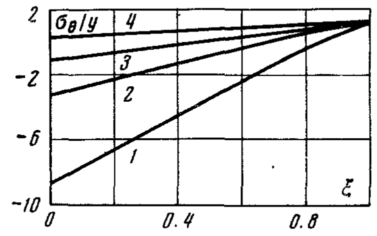

Fig. 2

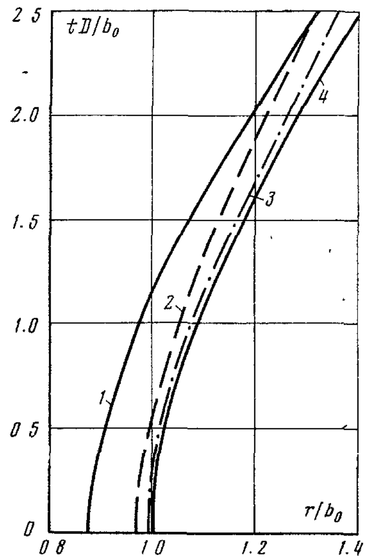

Fig. 3

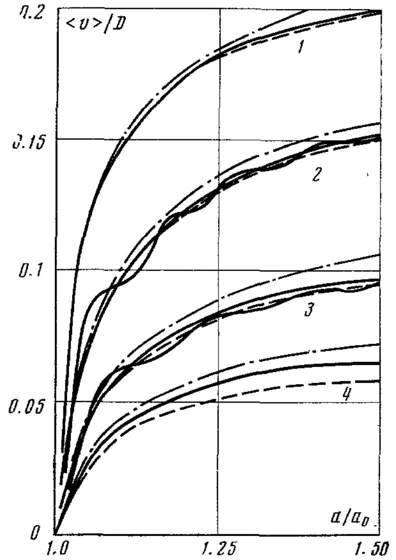

Fig. 4

We integrate (4) with respect to r from a up to the current value r and use the boundary condition: $\sigma_r = -p$ and $r = a$ to get

$$\sigma_r = -p + \varkappa Y \ln\frac{r}{a} + \gamma_0 (a\ddot{a} + \dot{a}^2)\ln\frac{r}{a} + \gamma_0\left(\frac{a^2\dot{a}^2}{2r^2} - \frac{\dot{a}^2}{2}\right) \tag{5}$$

We use the external boundary condition $\sigma_r = 0$ and $r = b$ to get

$$-p + \varkappa Y \ln\frac{b}{a} + \gamma_0 (a\ddot{a} + \dot{a}^2)\ln\frac{b}{a} + \gamma_0\left(\frac{a^2\dot{a}^2}{2b^2} - \frac{\dot{a}^2}{2}\right) = 0 \tag{6}$$

Let $p_0 = \rho_0 D^2/8$ be the pressure of the instantaneous detonation, with ρ_0 the explosive density and D the detonation rate. The condition for incompressibility is

$$b^2 - a^2 = b_0^2 - a_0^2$$

and then the law of equilibrium expansion (1) takes the form

$$p = p_0 (a_0 / a)^{2k} \tag{7}$$

where we introduce the dimensionless parameters

$$a' = \frac{a}{b_0}, \quad b' = \frac{b}{b_0}, \quad p' = \frac{p}{\rho_0 D^2}, \quad Y' = \frac{Y}{\rho_0 D^2}$$

$$\dot{a}' = \frac{\dot{a}}{D}, \quad \ddot{a}' = \frac{b_0 \ddot{a}}{D^2}, \quad t' = \frac{tD}{b_0}, \quad \chi = \frac{\gamma_0}{\rho_0}.$$

to get the nonlinear second-order differential equation

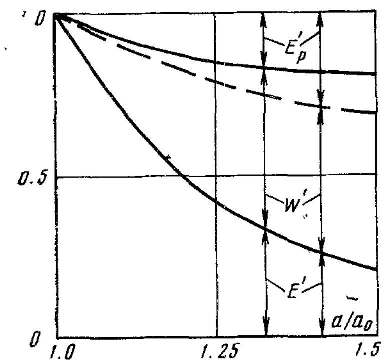

Fig. 5

TABLE 1

Shell material	b_c/b_0	b_f/b_0
Steel 45 Kh quenched	1.25	1.5
Steel 35	1.4	2.0
Armco	2.1	2.8
Cast iron VCh-60-2	1.25	1.5

$$\ddot{a} = B\dot{a}^2 + C,\ B = 1/2a \ln (H / a) - a /2H^2 \ln (H / a) - 1/a \qquad (8)$$
$$C = p_0 (a_0 / a)^{2k}/\chi\, a \ln (H / a) - \varkappa Y/a\chi, \quad H = (1 - a_0^2 + a^2)^{1/2}$$

Here and subsequently the primes to the dimensionless parameters are omitted.

The initial conditions are

$$a(0) = a_0,\ \dot{a}(0) = 0$$

Equation (8) was solved via a standard program using the Runge–Kutta method with various filling factors $\alpha = m/(m+M)$, where m is the mass of explosive and M is the mass of the shell; the various plasticity conditions were employed with different dynamic yield points. We assumed that $k=3$ in the expansion law.

Figure 2 shows the distribution of the tangential stresses over the thickness for various instants, where $\alpha = 0.2$, $Y = 10$ kbar, $\varkappa = 2/\sqrt{3}$, $\xi = (r - a)/(b - a)$; lines 1-4 correspond to dimensional times of 0, 1.7, 2.5, 3.2.

A shell subject to an explosive load has two zones; a zone adjoining the outer surface has a mixed state of stress ($\sigma_r < 0$, $\sigma_\theta > 0$) while the internal zone has a state of nonuniform hydrostatic compression. The zone boundary ($\sigma_\theta = 0$) moves as the shell expands and the pressure falls; Fig. 3 shows the motion of the internal and external boundaries (lines 1 and 4), and also that of the surfaces $\sigma_\theta = 0$ and $\sigma_r = \sigma_\theta$ (lines 2 and 3) for $\alpha = 0.4$, $Y = 10$ kbar, and the St. Venant plasticity condition.

As the stress distribution in thickness can be considered as linear to 7% at any instant, we have

$$\sigma_r = -p(b - r) / h, \quad \sigma_\theta = \varkappa Y - p(b - r) / h$$

where h is shell thickness.

The following is the depth of the stretched zone reckoned from the outer surface:

$$y = hY / p = y(t)$$

From (7) we get

$$y = hY (a / a_0)^{2k} / p_0$$

It is found [1] that failure in such a shell occurs when the outer zone has propagated through the entire thickness; the failure radius is defined by

$$a_f = a_0 (p_0 / Y)^{1/2k} \qquad (9)$$

It has been assumed inexplicitly in the derivation that the shell fails in the stretched zone by brittle tear.

If we assume that simultaneously the compressed zone fails by shear, so that failure has already occurred when the zone boundary arrives, the failure condition will be that the normal stresses in the sheared areas are zero, which corresponds to a state of pure shear:

$$\sigma_\theta = -\sigma_r,\ \sigma_z = 0,\ \sigma = (\sigma_r + \sigma_\theta + \sigma_z) / 3 = 0$$

An analogous conception is given in [3].

In that case, the failure radius is defined by

$$a^\circ = a_0 (2p_0 / Y)^{1/2k} \qquad (10)$$

We find that $a^\circ/a_f = 1.12$ for p_0, Y and $k = 3$.

Equations (9) and (10) do not incorporate the plastic properties of the material, such as the strain δ, shrinkage ψ, and so on.

Photographs have been taken of shells exploded in this way, and these show that one has to distinguish the radius b_c at which cracks arise at the outer surface and the failure radius b_f as recorded from the breakthrough of detonation products. Table 1 gives the relative values for these quantities (shell dimensions $a_0=10$ mm, $b_0=13.5$ mm, length $l_0=130$ mm, $a_0/b_0=0.74$, explosive TG 50/50).

The relative failure radius is 3.3 for copper shells ($a_0/b_0=0.83$).

Figure 4 shows the mean relative velocity $\langle v\rangle/D$ as a function of the relative radius a/a_0 for various α; the family of curves 1-4 corresponds to $\alpha=0.3$, 0.2, 0.1, 0.05, and the continuous monotonic lines represent results from (8) for the St. Venant plasticity condition and Y=10 kbar. The dot-and-dash lines show curves for an ideally plastic shell derived from

$$\frac{\langle v\rangle}{D}=\left\{\frac{\alpha}{8(1-\alpha)}\left[1-\left(\frac{a_0}{a}\right)^4\right]-\frac{2Y}{\gamma_0 D^2}\ln\frac{a}{a_0}\right\}^{1/2} \tag{11}$$

The dot-and-dash lines with points give results for the same formula with Y=0:

$$\frac{\langle v\rangle}{D}=\left\{\frac{\alpha}{8(1-\alpha)}\left[1-\left(\frac{a_0}{a}\right)^4\right]\right\}^{1/2} \tag{12}$$

Equation (12) does not incorporate the energy loss due to irreversible plastic deformation and gives high values.

Equation (11) gives underestimates for the velocity for α small, because it does not incorporate the actual stress distribution in the thickness.

The effects of the compressibility and elasticity on the integral characteristics have been examined by numerical integration of the equations of motion for an elastoplastic compressible shell with the same loading conditions (equilibrium expansion); we used the finite-difference scheme of [6].

Figure 4 shows $\langle v\rangle/D=f(a/a_0)$ for a shell with the elastic characteristics $G=0.81\cdot10^3$ kbar and $K=1.75\cdot10^3$ kbar, with Y= 10 kbar and parameters for the fault compressibility in accordance with [7] for two filling factors (solid lines, nonmonotonic). It is clear that the compressibility and elasticity have only small effects on the final velocities under these conditions.

The energy redistribution for an incompressible shell is indicated by Fig. 5, where

$$E'=E/E_0,\ W'=W/E_0,\ E_p'=E_p/E_0$$

with E_0 and E the initial and current internal energies of the detonation products, W the kinetic energy of the shell, and E_p the energy of plastic shape change. The graph has been constructed in the form

$$E'+W'+E_p'=1$$

Figure 5 shows results corresponding to Y= 10 kbar (solid line) and Y=20 kbar (broken line). In both cases, $\alpha=0.05$, and the St. Venant plasticity condition was used.

It has been proposed [3] that thermoplastic shear may occur in such a shell, so we estimated the temperature arising from the plastic deformation. The rise was not more than 150-200° for $a/a_0=1.5$ and 10-20 kbar for the yield point, which should not substantially affect the properties. This estimate does not incorporate the direct heat transfer to the shell from the detonated products.

LITERATURE CITED

1. G. I. Taylor, Sci. Papers of G. I. Taylor, 8, Cambridge Univ. Press (1963).
2. N. F. Mott, "Fragmentation of shell cases," Proc. Roy. Soc. London, Ser. A., 189, No. 1018 (1947).
3. G. R. Hoggat and R. F., Recht, "Fracture behavior of tubular bombs," J. Appl. Phys., 39, No. 3 (1968).
4. S. T. S. Al-Hassani, H. Hopkins, and W. Johnson, "A note on the fragmentation of tubular bombs," Internat. J. Mech. Sci., 11, No. 6 (1969).
5. J. Pearson and J. Reinhart, "Deformation and failure of thick-walled steel cylinders on explosion," in: Mechanics, a Collection of Translations and Surveys of the Foreign Periodical Literature [in Russian], No. 3 (1953).
6. A. V. Kashivskii, Yu. V. Korovin, and V. A. Odintsov, "Motion of a shell on axial detonation," Prikl. Mekhan. Tekh. Fiz., No. 1 (1971).
7. H. M. Sternberg and D. Piacesi, "Interaction of oblique detonation waves with iron," Phys. Fluids, 9, No. 7 (1966).